

Unparticle Physics And Gauge Coupling Unification

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Abstract

Unparticle physics from a hidden conformal sector can alter the evolution of the Standard Model (SM) gauge couplings via TeV scale threshold corrections. We discuss how this may lead to gauge coupling unification at $M_{GUT} \approx 2 \times 10^{15} \text{ GeV} - 5 \times 10^{17} \text{ GeV}$ without introducing new particles in the SM sector.

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It has been recognized for a long time that with a canonical normalization of $5/3$ for $U(1)_Y$, the three SM gauge couplings fail to unify without introducing new physics, which often means the introduction of some new particles between the electroweak scale and the unification scale M_{GUT} . The minimal supersymmetric standard model (MSSM) provides the most compelling example of this approach with $M_{\text{GUT}} \approx 2 \times 10^{16}$ GeV [1]. An alternative scenario in which no new particles are introduced, can be realized from higher dimensional GUTs in which the canonical normalization for $U(1)_Y$ gauge coupling is replaced by a different choice, say $4/3$, which leads to gauge coupling unification at M_{GUT} close to 4×10^{16} GeV [2]. In some other modifications of the SM involving low energy supersymmetry plus additional 'matter' fields, unification of the gauge couplings at the string scale ($\sim 5 \times 10^{17}$ GeV) can be realized [3].

In this paper we propose to achieve unification of the SM gauge couplings by exploiting the so-called "unparticle physics" sector introduced in [4]. We find that threshold corrections via unparticle physics at the TeV scale can succeed in unifying the three SM gauge couplings with the conventional $5/3$ normalization for $U(1)_Y$, and without introducing any new particles. Depending on the size of these threshold corrections for each SM gauge coupling, the unification scale can vary by a few order of magnitudes around 10^{15} GeV. These corrections may be revealed by precision measurements of the SM gauge couplings around the TeV scale at future high energy collider experiments such as the LHC and the International Linear Collider (ILC).

The basic structure of the unparticle physics is as follows. First, we introduce a coupling between a new SM singlet operator (\mathcal{O}_{UV}) with dimension d_{UV} and a SM operator \mathcal{O}_{SM} with dimension n ,

$$\mathcal{L} = \frac{c_n}{M^{d_{\text{UV}}+n-4}} \mathcal{O}_{\text{UV}} \mathcal{O}_{\text{SM}}, \quad (1)$$

where c_n is a dimensionless constant, and M is the energy scale characterizing the new physics. This new physics sector is assumed to become strong and conformal at some energy $\Lambda_{\mathcal{U}}$, and the operator \mathcal{O}_{UV} flows to the unparticle operator \mathcal{U} with dimension $d_{\mathcal{U}}$. In the low energy effective theory, we obtain an interaction given by

$$\mathcal{L} = c_n \frac{\Lambda_{\mathcal{U}}^{d_{\text{UV}}-d_{\mathcal{U}}}}{M^{d_{\text{UV}}+n-4}} \mathcal{U} \mathcal{O}_{\text{SM}} \equiv \frac{1}{\Lambda^{d_{\mathcal{U}}+n-4}} \mathcal{U} \mathcal{O}_{\text{SM}}, \quad (2)$$

where the unparticle dimension $d_{\mathcal{U}}$ is determined at the scale $\Lambda_{\mathcal{U}}$ (which is induced through dimensional transmutation), and Λ is the (effective) cutoff scale of the low energy effective theory. In this paper, we consider only a "scalar" unparticle and we first focus on the interaction between the unparticle and the SM Higgs sector given by [5], [6], [7], [8],

$$\mathcal{L} = \frac{1}{\Lambda^{d_{\mathcal{U}}+n-4}} \mathcal{U} \mathcal{O}_{\text{SM}}(H^\dagger H), \quad (3)$$

where H is the SM Higgs doublet and $\mathcal{O}_{\text{SM}}(H^\dagger H)$ is a SM operator given as a function of the gauge invariant combination $H^\dagger H$. Once the Higgs doublet develops a vacuum expectation value (VEV), a tadpole term for the unparticle is induced,

$$\mathcal{L}_{\mathcal{U}} = \Lambda_{\mathcal{U}}^{4-d_{\mathcal{U}}} \mathcal{U}, \quad (4)$$

where $\Lambda_{\mathcal{U}}^{4-d_{\mathcal{U}}} = \langle \mathcal{O}_{\text{SM}} \rangle / \Lambda^{d_{\mathcal{U}}+n-4}$. Through this tadpole term, the unparticle acquires a VEV and the conformal symmetry is broken [5]. This VEV is given by

$$\langle \mathcal{U} \rangle \lesssim \Lambda_{\mathcal{U}}^{d_{\mathcal{U}}}. \quad (5)$$

With the cutoff scale of the effective theory around a TeV, we expect $\langle \mathcal{U} \rangle \approx 100 \text{ GeV} - 1 \text{ TeV}$.

Next we introduce couplings between the unparticle and the SM gauge bosons as follows [5]:

$$\mathcal{L}_{\mathcal{U}} = -\frac{\lambda_3}{4g_3^2} \frac{\mathcal{U}}{\Lambda^{d_{\mathcal{U}}}} G_{\mu\nu}^A G^{A\mu\nu} - \frac{\lambda_2}{4g_2^2} \frac{\mathcal{U}}{\Lambda^{d_{\mathcal{U}}}} F_{\mu\nu}^A F^{A\mu\nu} - \frac{\lambda_1}{4g'^2} \frac{\mathcal{U}}{\Lambda^{d_{\mathcal{U}}}} B_{\mu\nu} B^{\mu\nu}, \quad (6)$$

where $G_{\mu\nu}^A$, $F_{\mu\nu}^A$ and $B_{\mu\nu}$ denote the field strengths for the SM gauge group $\text{SU}(3) \times \text{SU}(2) \times \text{U}(1)_Y$, and λ_i are dimensionless coefficient of order unity or less. For $\langle \mathcal{U} \rangle \neq 0$, Eq. (6) leads to modifications of the gauge kinetic terms. (This is reminiscent of some early work [9] based on modification of gauge kinetic energy terms in GUTs). Thus

$$\mathcal{L} = -\frac{1}{4g_i^2} \left[1 + \lambda_i \frac{\langle \mathcal{U} \rangle}{\Lambda^{d_{\mathcal{U}}}} \right] \mathcal{F}_{\mu\nu} \mathcal{F}^{\mu\nu} \simeq -\frac{1}{4g_i^2} (1 + \epsilon_i) \mathcal{F}_{\mu\nu} \mathcal{F}^{\mu\nu}, \quad (7)$$

where $\mathcal{F}_{\mu\nu}$ represents the appropriate SM field strength, and

$$\epsilon_i \equiv \lambda_i \frac{\langle \mathcal{U} \rangle}{\Lambda^{d_{\mathcal{U}}}} \simeq \lambda_i \left(\frac{\Lambda_{\mathcal{U}}}{\Lambda} \right)^{d_{\mathcal{U}}}. \quad (8)$$

This modification can be interpreted as a threshold correction in the gauge coupling evolution across the scale $\langle \mathcal{U} \rangle^{1/d_{\mathcal{U}}} \sim \Lambda_{\mathcal{U}}$ [5].

For $\Lambda_{\mathcal{U}} \lesssim M_Z$, the ϵ 's are severely constrained by the current precision measurements on the fine structure constant [5]. The evolution of the fine structure constant from low energy to the Z-pole (M_Z) is consistent with the SM, and the largest uncertainty arises from the fine structure constant measured at the Z pole [11],

$$\alpha_{em}^{-1}(M_Z) = 127.918 \pm 0.019. \quad (9)$$

This uncertainty (in the $\overline{\text{MS}}$ scheme) can be converted to the constraint

$$\epsilon \lesssim 1.4 \times 10^{-4}. \quad (10)$$

Here, ϵ is an admixture of ϵ_1 and ϵ_2 corresponding to the QED coupling. In the following, we consider the case $\Lambda_{\mathcal{U}} > M_Z$, with the expectation that the gauge coupling evolution in this region can be precisely measured in future experiments in order to test this scenario. From Eq. (7), the threshold correction for each gauge coupling at $\Lambda_{\mathcal{U}}$ is given by

$$\Delta g_i^2(\Lambda_{\mathcal{U}}) = g_i^2(\Lambda_{\mathcal{U}}) \times \epsilon_i. \quad (11)$$

Since ϵ_i are free parameters, we can regard the threshold corrections as theoretical ambiguities of the SM gauge couplings at $\Lambda_{\mathcal{U}}$ associated with the unparticle physics. We are assuming here that the three ϵ_i are all distinct. The TeV scale unparticle physics is not "aware" of the underlying grand unified theory at M_{GUT} . One way to realize this is to consider a five dimensional GUT compactified on S^1/Z_2 such that only the SM gauge symmetry survives at one of the fixed points. The couplings in Eq. (6) can be realized on this fixed point, with the conformal sector restricted to the 4D brane (fixed point).

Let us now see how the threshold corrections via unparticle physics enable the SM gauge couplings to unify at M_{GUT} without introducing any new particles or non-canonical normalization for the $U(1)_Y$ gauge coupling. We employ two-loop renormalization group equations (RGE) for the running gauge couplings [10],

$$\frac{dg_i}{d \ln \mu} = \frac{b_i}{16\pi^2} g_i^3 + \frac{g_i^3}{(16\pi^2)^2} \sum_{j=1}^3 B_{ij} g_j^2, \quad (12)$$

where μ is the renormalization scale, g_i ($i = 1, 2, 3$) are the SM gauge couplings and

$$b_i = \left(\frac{41}{10}, -\frac{19}{6}, -7 \right), \quad b_{ij} = \begin{pmatrix} \frac{199}{50} & \frac{27}{10} & \frac{44}{5} \\ \frac{9}{10} & \frac{33}{6} & 12 \\ \frac{11}{10} & \frac{6}{2} & -26 \end{pmatrix}, \quad (13)$$

with $(\alpha_1, \alpha_2, \alpha_3) = (0.01681, 0.03354, 0.1176)$ at the Z-pole (M_Z) [11].

Figure 1 shows the evolution of g_i (more precisely of $\alpha_i^{-1} = 4\pi/g_i^2$), after incorporating the threshold corrections, with $|\epsilon_i| \leq 0.1$ as an example, at $\Lambda_{\mathcal{U}} = 1$ TeV. We show in Figure 2 the result for $\epsilon_1 = -0.05$ and $\epsilon_2 = \epsilon_3 = 0.1$. Unification of the three gauge couplings is achieved at $M_{\text{GUT}} = 2 \times 10^{15}$ GeV. The parameter set in Figure 3, $\epsilon_1 = -0.18$, $\epsilon_2 = 0.2$ and $\epsilon_3 = 0.1$, realizes gauge coupling unification at $M_{\text{GUT}} = 5 \times 10^{17}$ GeV (string scale). Finally, just as in the MSSM, gauge coupling unification at $M_{\text{GUT}} = 2 \times 10^{16}$ GeV is possible, as shown in Figure 4. Intermediate scale unification with unparticle has been discussed in [12].

Baryon number is normally broken as a result of unification of quarks and leptons in GUT multiplets, and proton decay mediated by superheavy gauge bosons is a typical prediction. Non-observation of proton decay in current experiments leads to the bound $M_{\text{GUT}} \gtrsim 2 \times 10^{15}$ GeV on M_{GUT} [14].

In summary, we have considered an unparticle sector which couples to the SM Higgs sector. In this case, the Higgs VEV triggers not only the electroweak symmetry breaking but also the conformal symmetry breaking in the unparticle sector. As a consequence, the unparticle naturally develops a VEV which, through couplings to the SM gauge bosons, works as threshold corrections in the evolution of the SM gauge couplings across the conformal symmetry breaking scale. The threshold corrections alter the SM gauge couplings at high energies. We have examined the possibility of gauge coupling unification through these threshold corrections with only the SM particle contents. We have shown that this unification can be successfully realized with threshold corrections of order 10% for the SM gauge couplings α_i . Such threshold correction may be revealed through precision measurements in future experiments at high energies.

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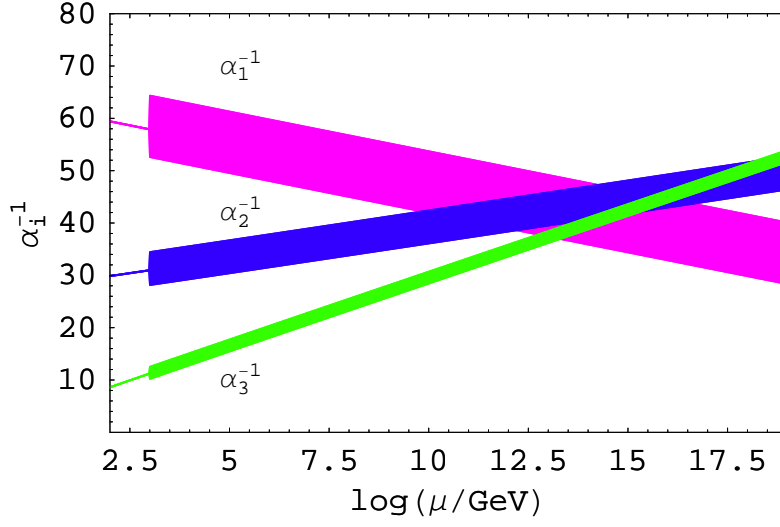


Figure 1: Evolution of the three SM gauge couplings α_i^{-1} with unparticle threshold corrections, $|\epsilon_i| \leq 0.1$, at 1 TeV.

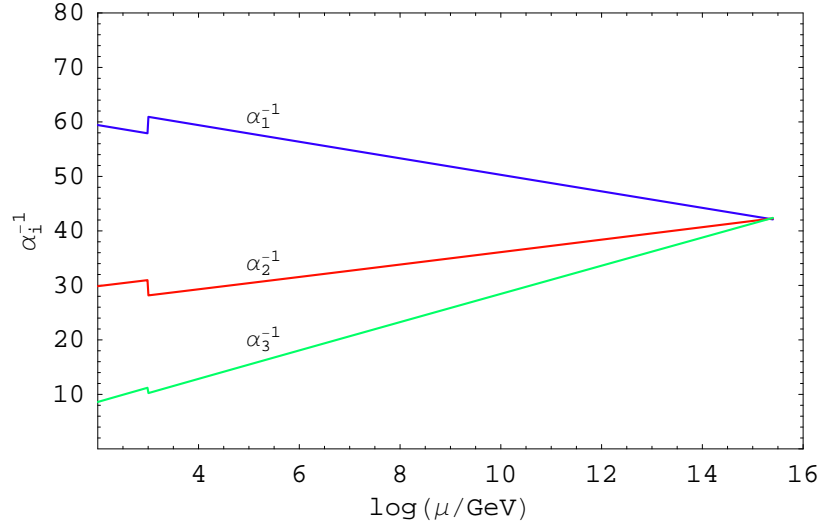


Figure 2: Evolution of α_i^{-1} with unparticle threshold corrections at 1 TeV. Here, $\epsilon_1 = -0.05$ and $\epsilon_2 = \epsilon_3 = 0.1$. Unification occurs at $M_{\text{GUT}} = 2 \times 10^{15}$ GeV.

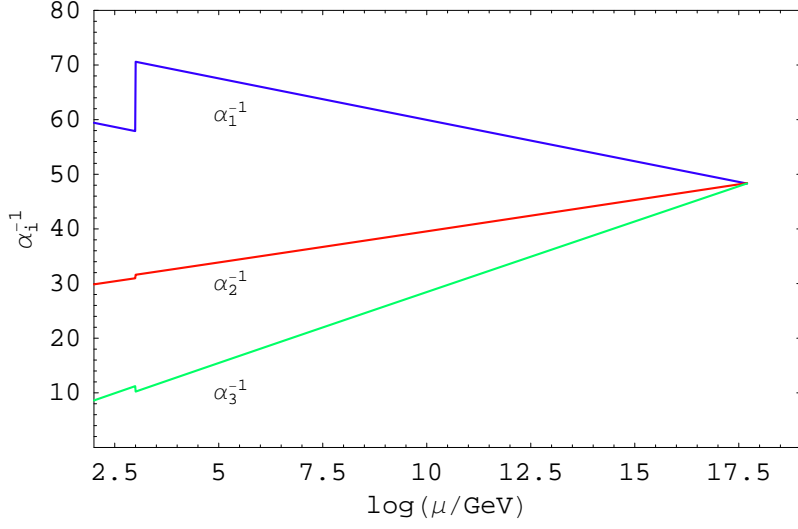


Figure 3: Evolution of α_i^{-1} with unparticle threshold corrections at 1 TeV. Here, $\epsilon_1 = -0.18$, $\epsilon_2 = -0.02$, and $\epsilon_3 = 0.1$. The three gauge couplings unify at $M_{\text{GUT}} = 5 \times 10^{17}$ GeV.

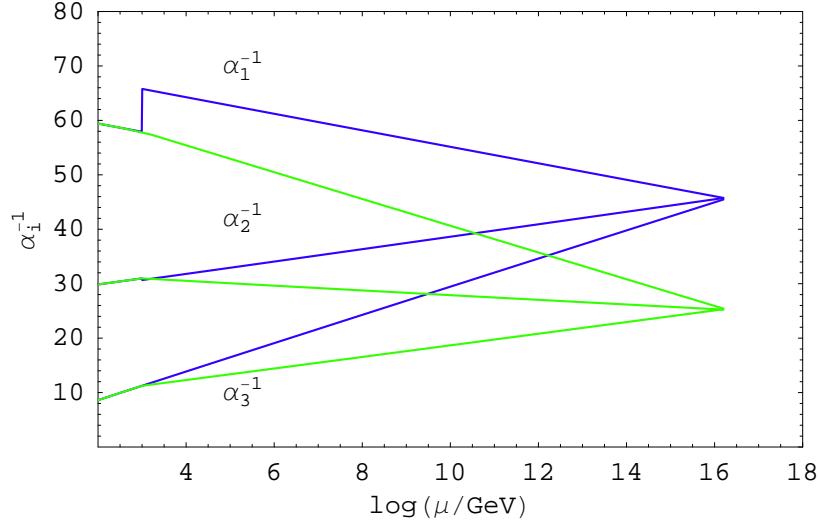


Figure 4: Evolution of α_i^{-1} with unparticle threshold corrections, $\epsilon_1 = -0.12$, $\epsilon_2 = 0.01$ and $\epsilon_3 = 0$, at 1 TeV (blue lines). Green lines show evolution of α_i^{-1} in MSSM with the SUSY scale at 1 TeV.